

The contribution of Etalon I and II to the Earth Orientation determination

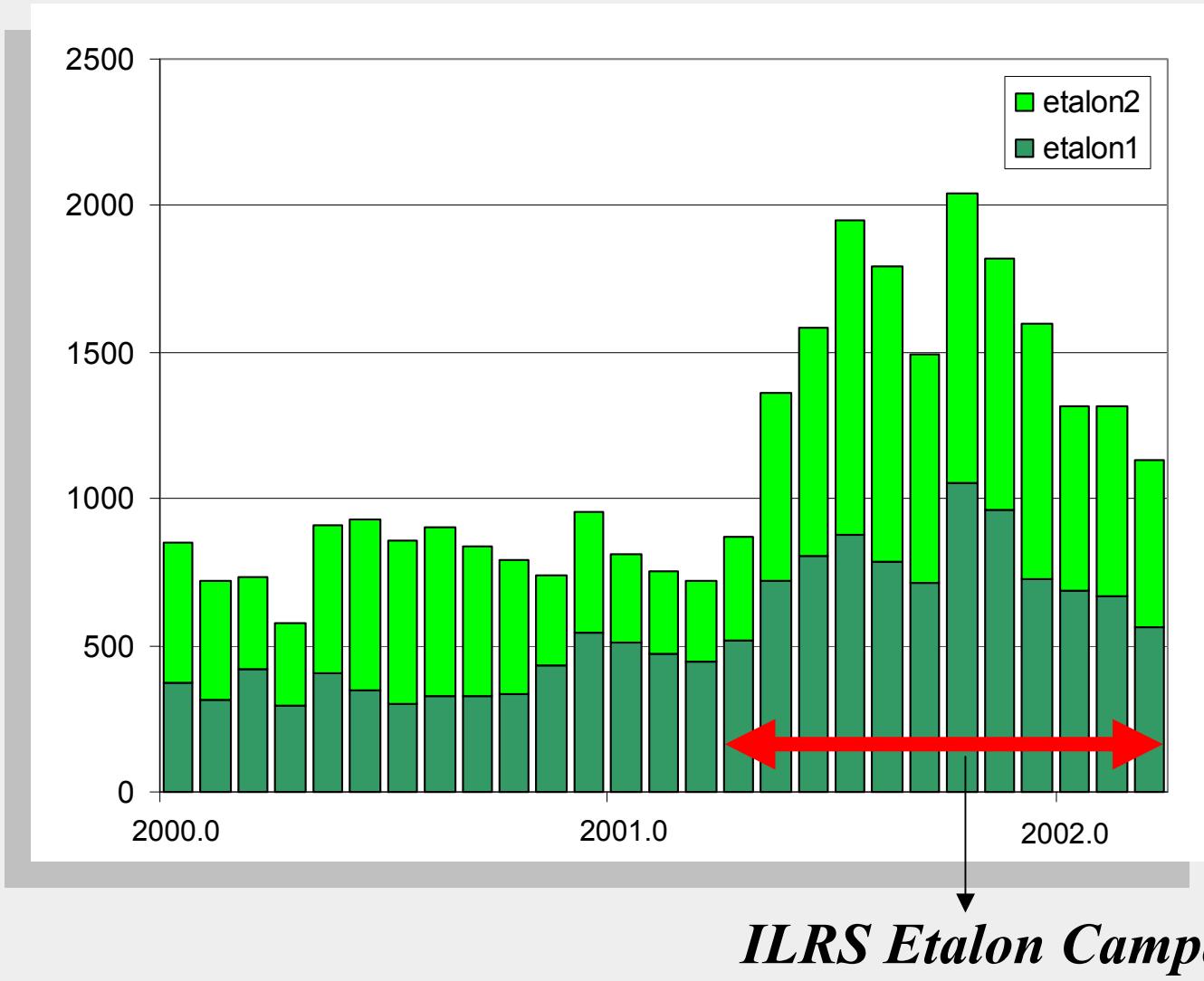
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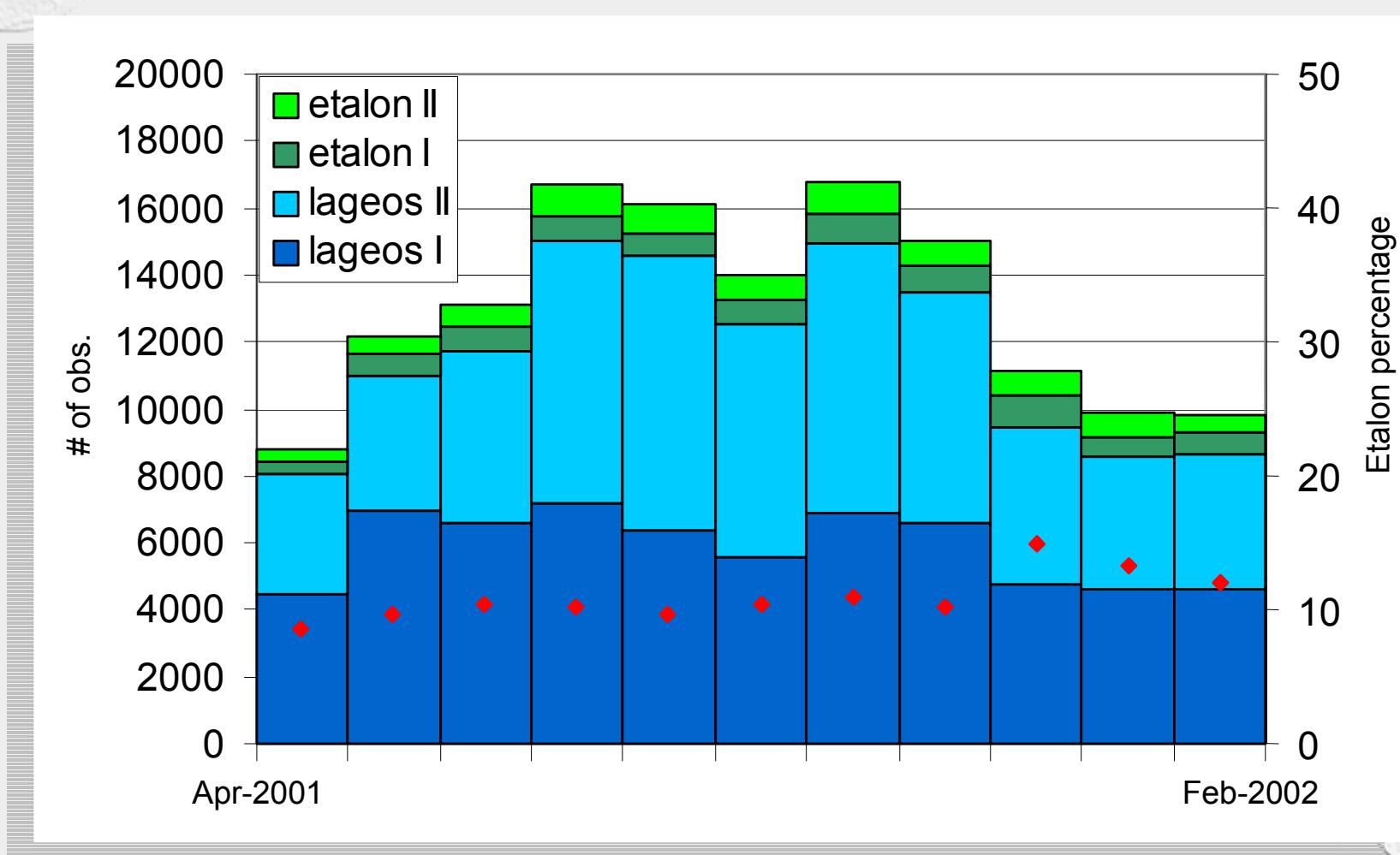
XXVII EGS General Assembly

Nice, April 22-26, 2002

Etalon observations



Analysed data set





Arc definition & Processing

- Arcs have been reduced using iterated bayesian least squares (Geodyn).
- Then each group of Lageos I-II and Etalon I-II arc-normal-matrices have been formatted into a single matrix and inverted in a least squares sense (Solve).

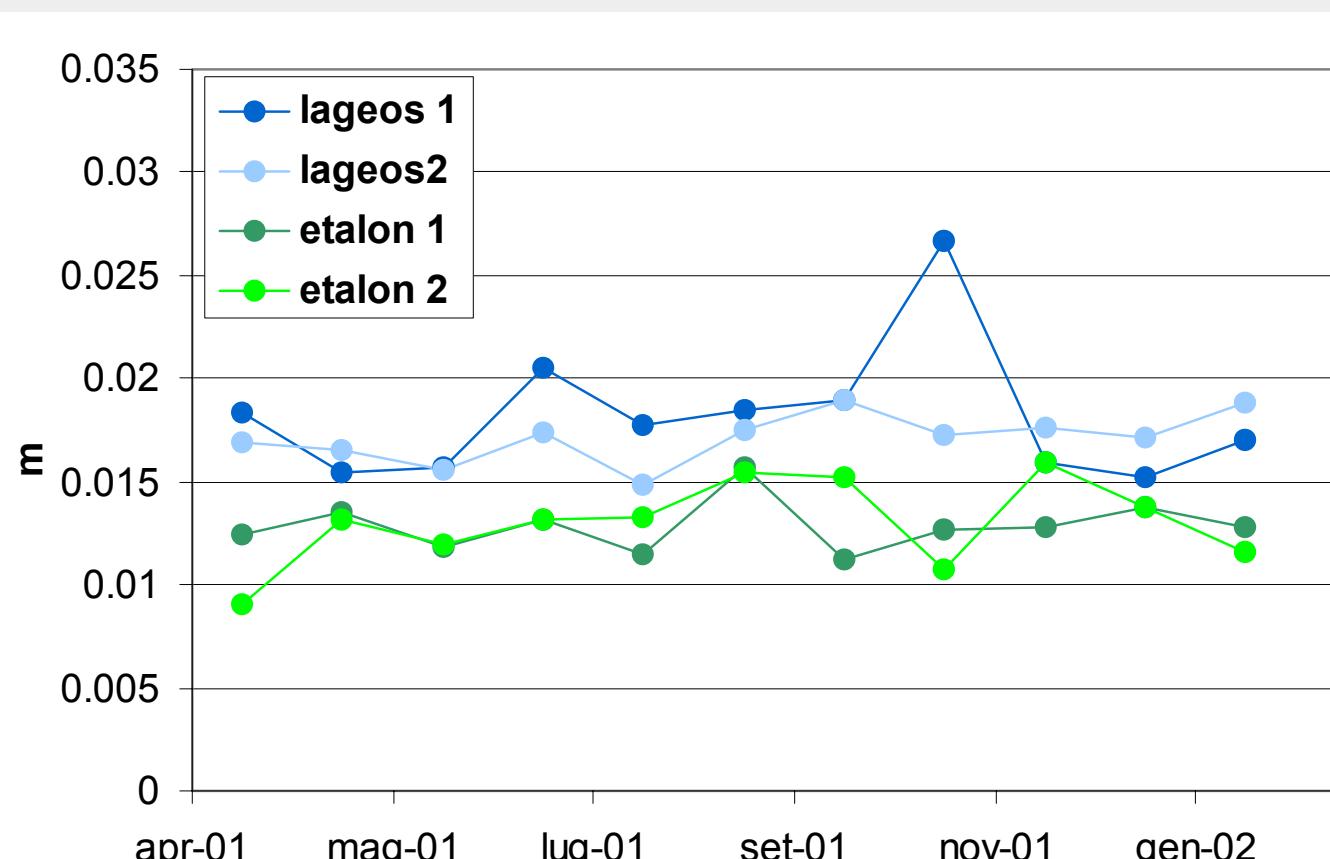
Estimated Parameters

- 4-weekly station coordinates
- daily EOP and EOP rate
- 4-weekly range biases
- state vectors
- weekly constant along-track accel.
- weekly once-per-rev along-track accel.
- weekly once-per-rev cross-track accel.

A priori values

- ITRF2000 for coordinates propagated at the arc epoch time
- EOPC04
- biases: 0
- const. along-track acc: L1 $-0.3060 \cdot 10^{-11}$
L2 $-0.2569 \cdot 10^{-11}$
- once-per-rev cross-track acc: 0
- once-per-rev radial-track acc: 0

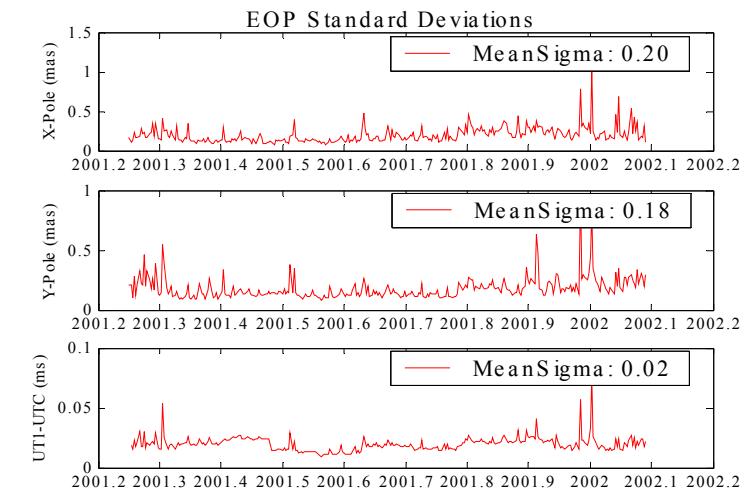
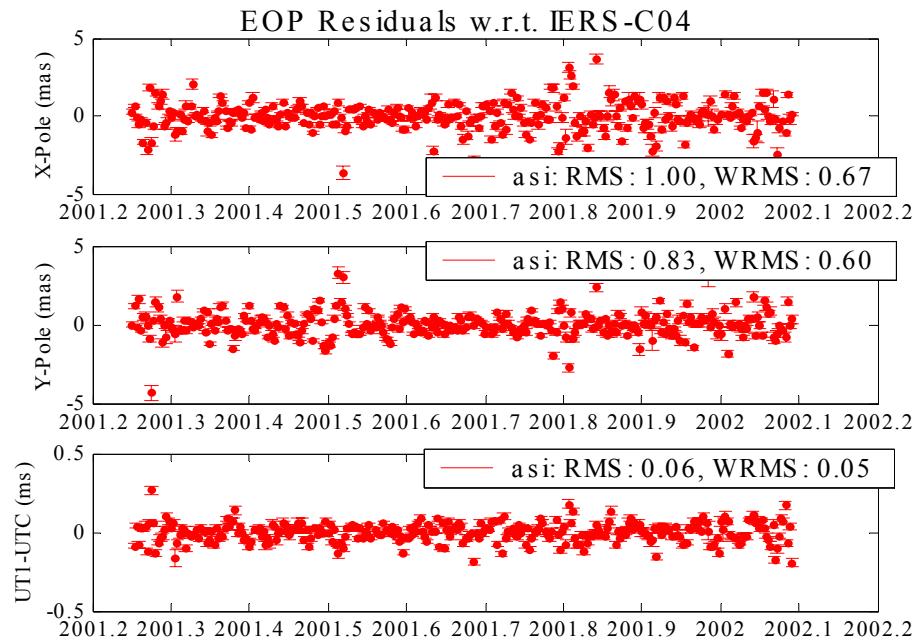
ARC Residual WRMS



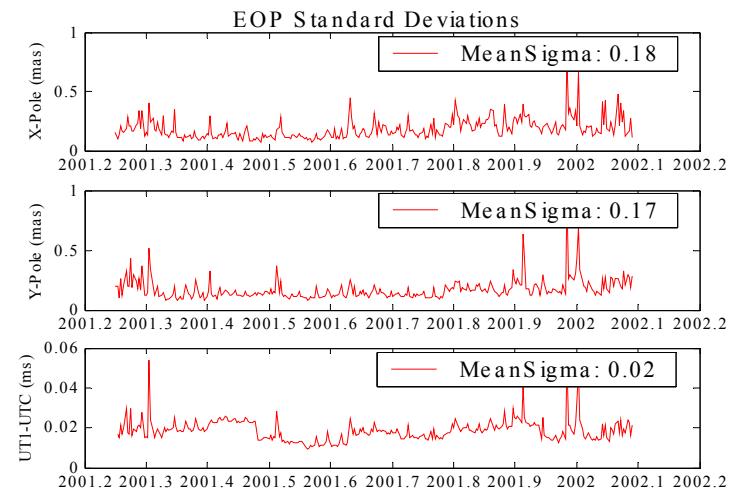
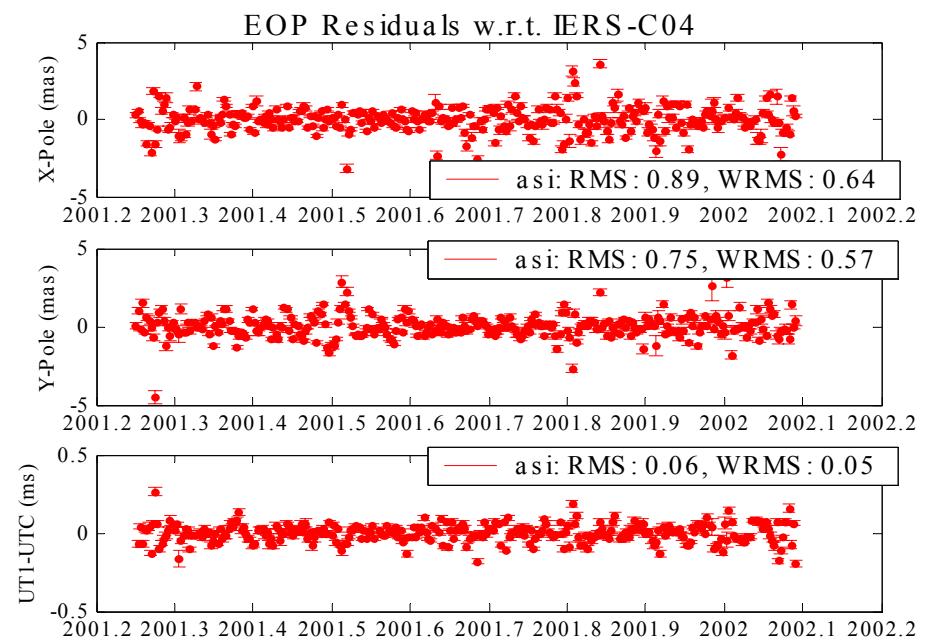


The ASI/CGS solution

Lageos



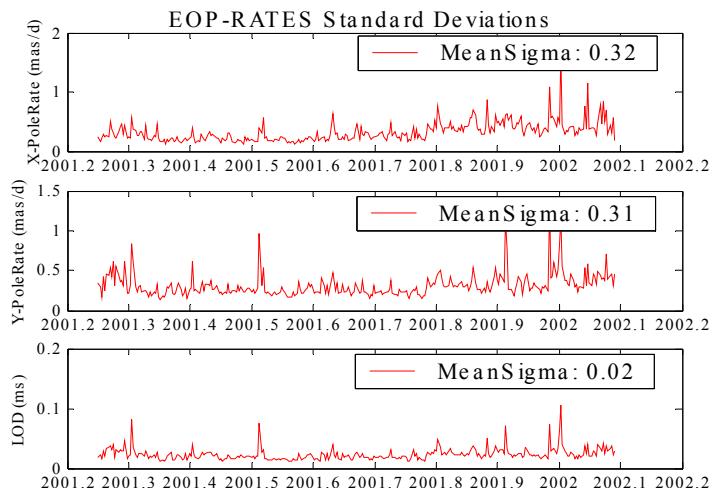
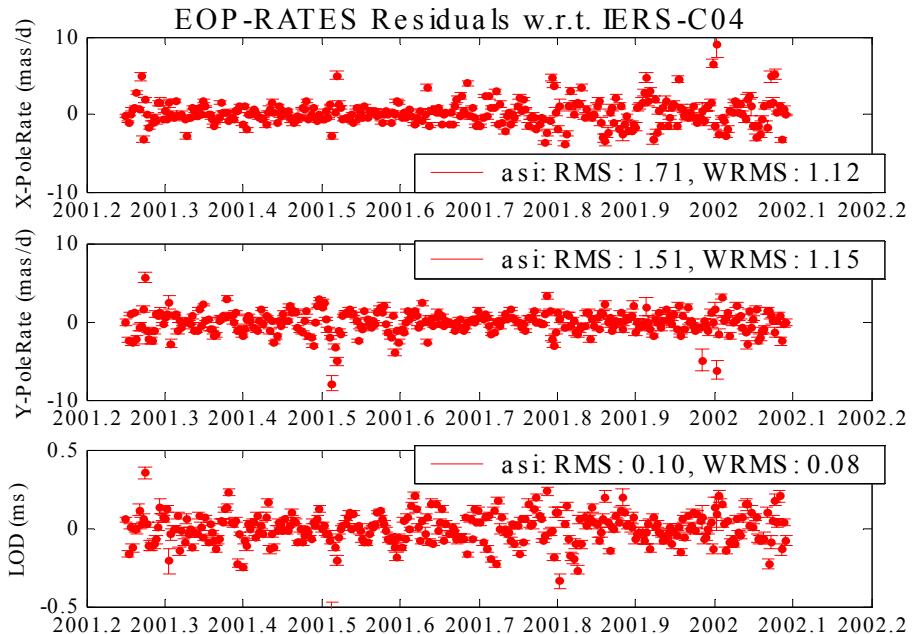
Lageos & Etalon



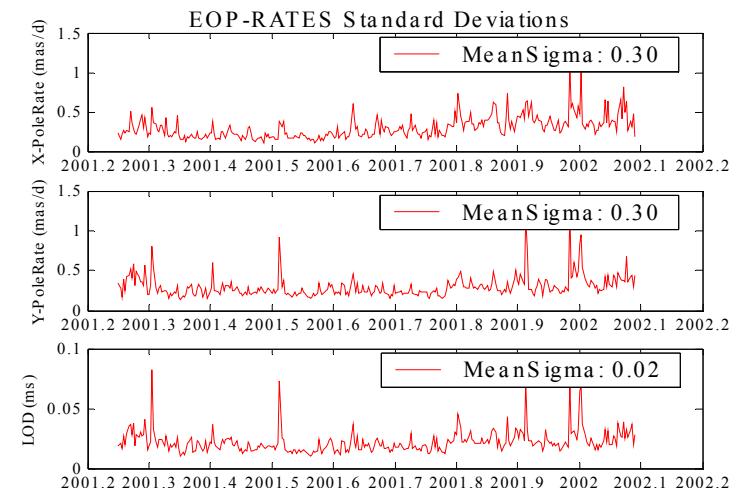
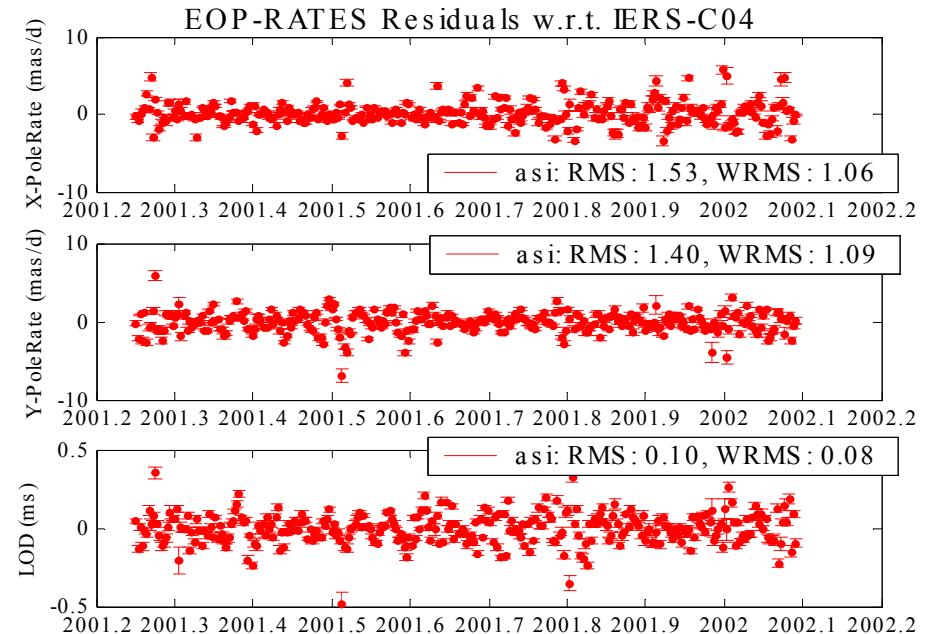


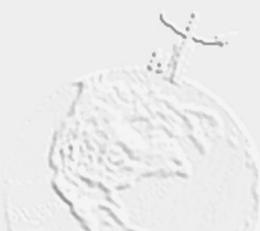
The ASI/CGS solution

Lageos



Lageos & Etalon





The ILRS AWG pilot project solutions

EOP

		X (mas)		Y (mas)	
		wrms *	σ	wrms *	σ
ASI L	0.67	0.20	0.60	0.18	
	0.64	0.18	0.57	0.17	
AUS L	0.50	0.07	0.60	0.07	
	0.50	0.07	0.57	0.07	
BKG L	0.25	0.38	0.29	0.37	
	0.25	0.38	0.28	0.36	
CRL L	0.42	0.12	0.46	0.11	
	0.39	0.12	0.40	0.10	
NERC L	0.32	0.13	0.37	0.12	
	0.30	0.13	0.36	0.12	
JCET L	0.41	0.18	0.40	0.16	
	0.43	0.18	0.43	0.17	

EOP rate

		Xr (mas/d)		Yr (mas/d)		LOD (ms/d)	
		wrms *	σ	wrms *	σ	wrms *	σ
ASI L	1.12	0.32	1.15	0.31	0.08	0.02	
	1.06	0.30	1.09	0.30	0.08	0.02	
NERC L	1.87	0.42	1.77	0.50	0.26	0.04	
	1.77	0.41	1.72	0.48	0.26	0.04	
JCET L	0.68	0.29	0.76	0.29	0.15	0.05	
	0.71	0.29	0.82	0.30	0.11	0.03	

L = Lageos solution
L+E = Lageos&Etalon solution

* wrms of the residuals with respect to EOPC04



The Least Squares error budget

from Linear Least Squares:

$$E(\vec{M}) = A\vec{\theta}$$

estimated parameters $\vec{\theta} = (A^T W A)^{-1} A^T W \vec{M}$

Covariance $V(\theta) = (A^T W A)^{-1}$

The **Dilution Of Precision** factor:

$$\text{DOP}_{\text{factor}} \equiv \sqrt{\text{trace}\left[\left(A^T A\right)^{-1}\right]}$$

weight matrix $W_{(n \times n)} = \begin{pmatrix} \sigma_1^2 & \dots & \text{cov} \\ \vdots & \ddots & \vdots \\ \text{cov} & \dots & \sigma_n^2 \end{pmatrix}^{-1}$

design matrix $A_{(n \times k)} = \begin{pmatrix} \frac{\partial m_1}{\partial \theta_1} & \dots & \frac{\partial m_1}{\partial \theta_k} \\ \vdots & \ddots & \vdots \\ \frac{\partial m_n}{\partial \theta_1} & \dots & \frac{\partial m_n}{\partial \theta_k} \end{pmatrix}$

vector of estimated parameters $\vec{\theta}_{(k \times 1)} = \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_k \end{pmatrix}$

vector of measurements $\vec{M}_{(n \times 1)} = \begin{pmatrix} m_1 \\ \vdots \\ m_n \end{pmatrix}$



The DOP factor: a tool for the analysis design

For independent, equally weighted measurements the diagonal DOP factors are the conversion factors between the measurement error and the parameter error.

$$\text{If } (\sigma_1 = \dots = \sigma_n = \sigma_0) \quad \text{the} \quad V(\theta) = (A^T A)^{-1} \sigma_0^2 = (\text{DOP})^2 \sigma_0^2$$

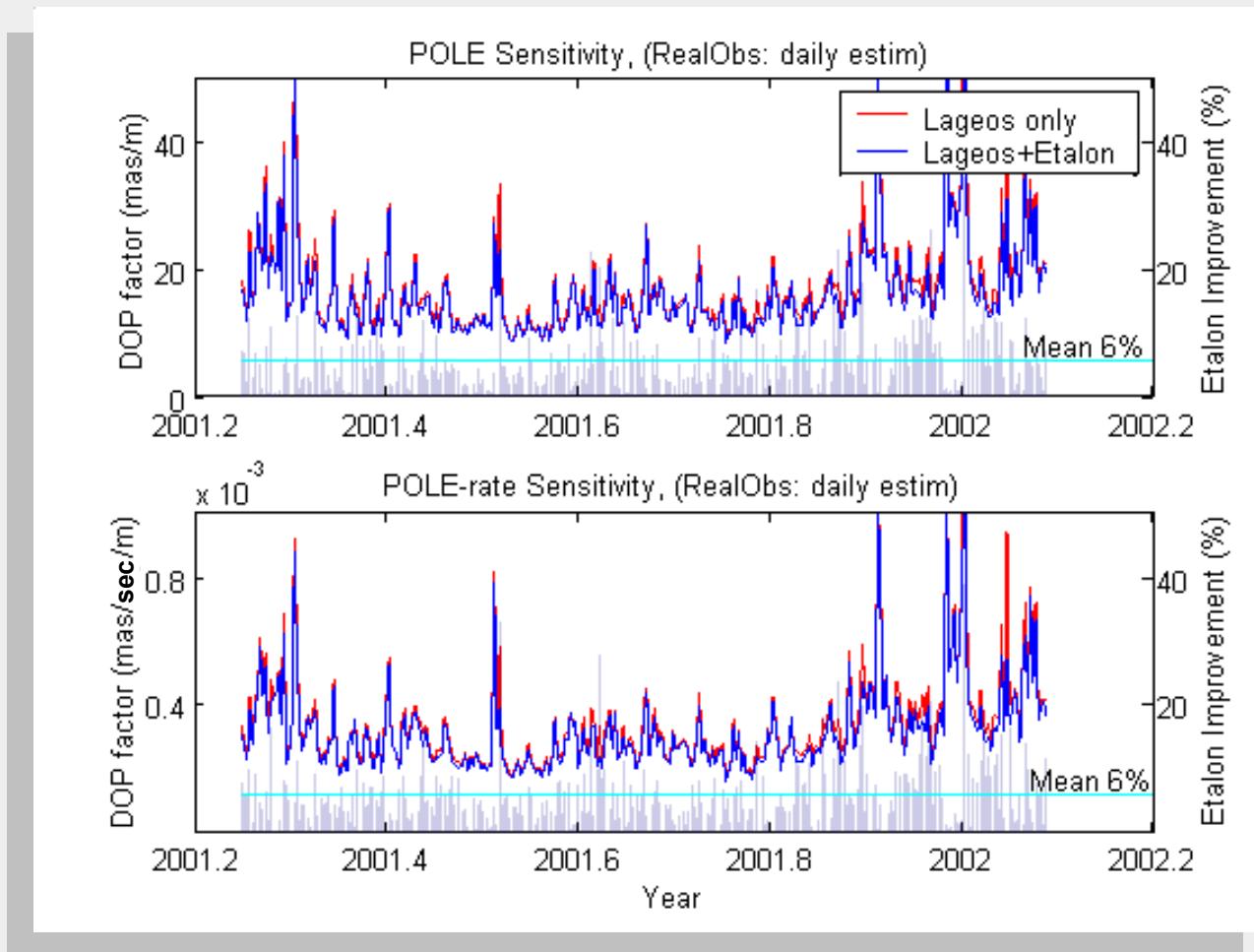
The DOP factor measures the dilution of the measurement error. The goal is to get the **minimum** DOP.

$$\sigma(\theta_k) = (\text{DOP})_k \sigma_0$$

The DOP factors for EOP tell us how well (in a "geometrical" sense) the observations can recover the Earth Orientation. In designing an experiment the DOP factor should be minimized.

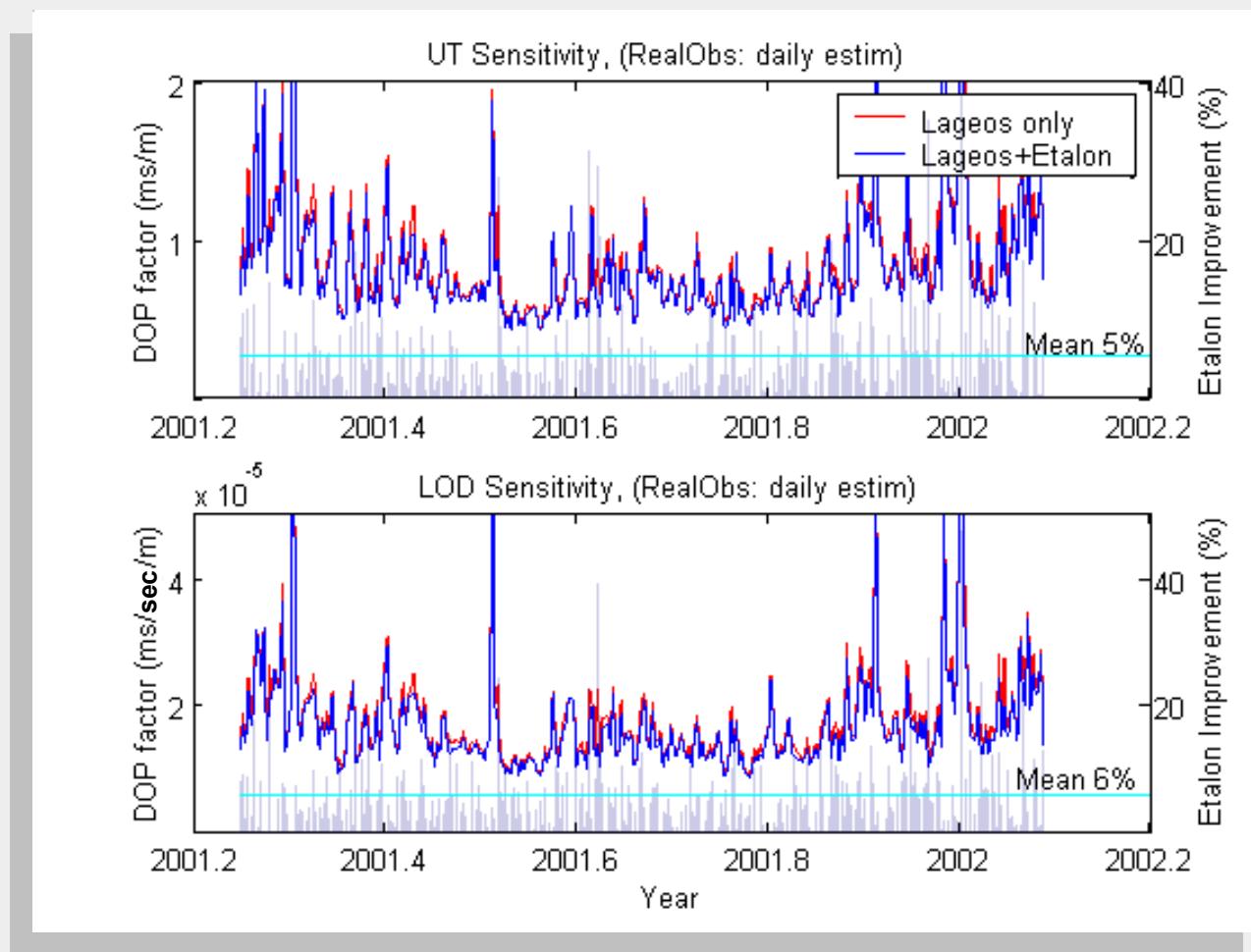


The EOP DOP from real observations





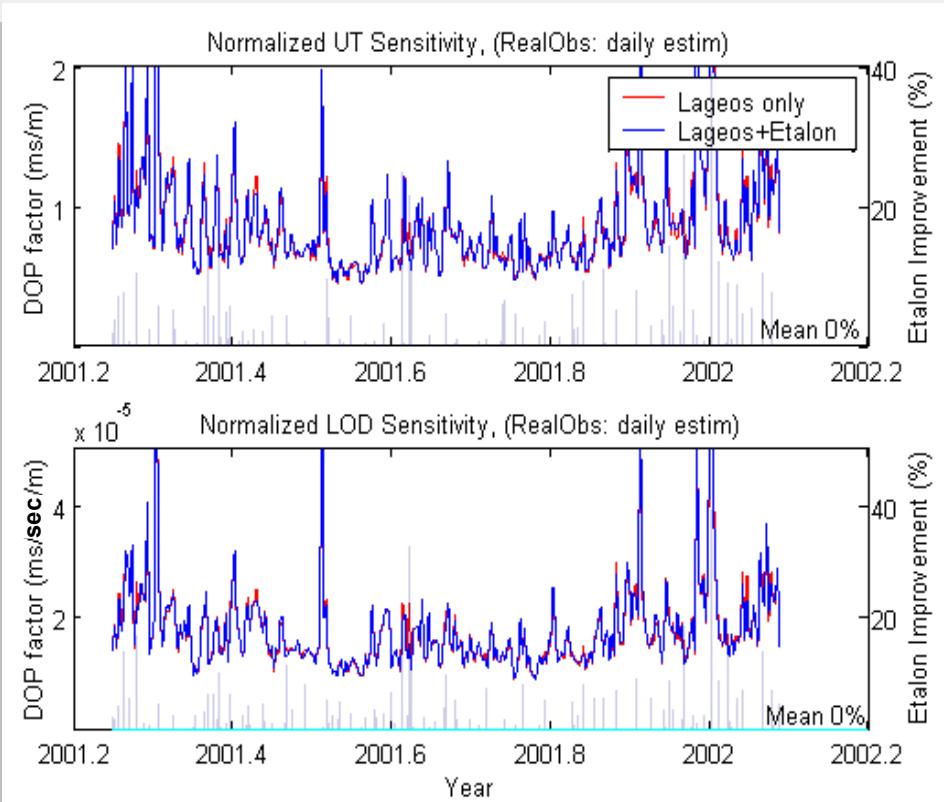
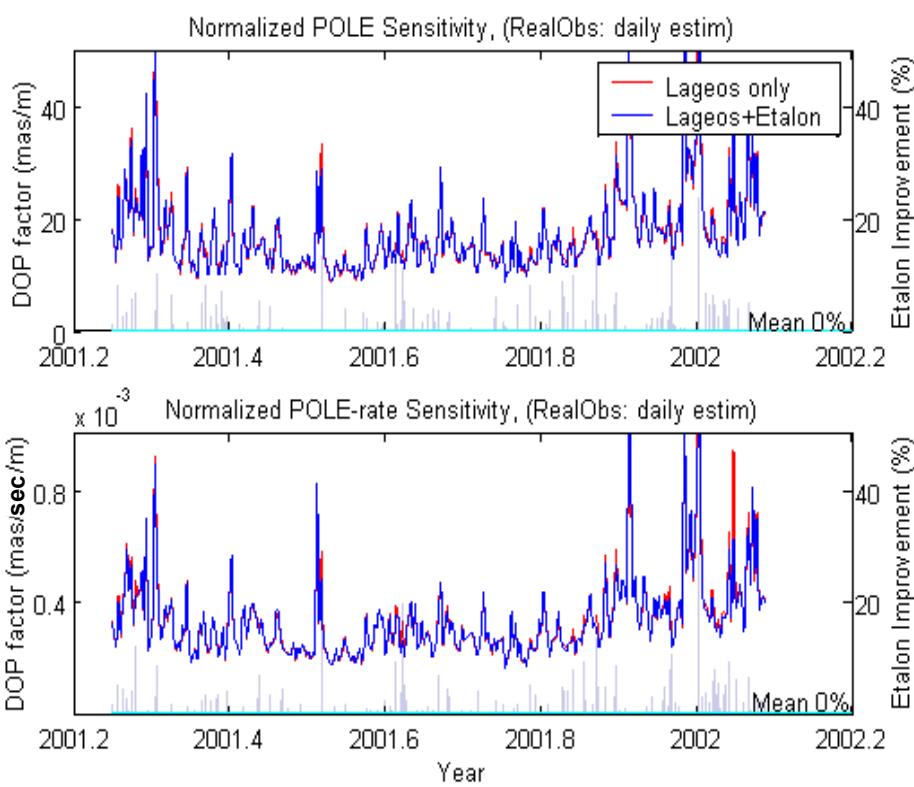
The EOP DOP from real observations



The normalized EOP DOP from real observations

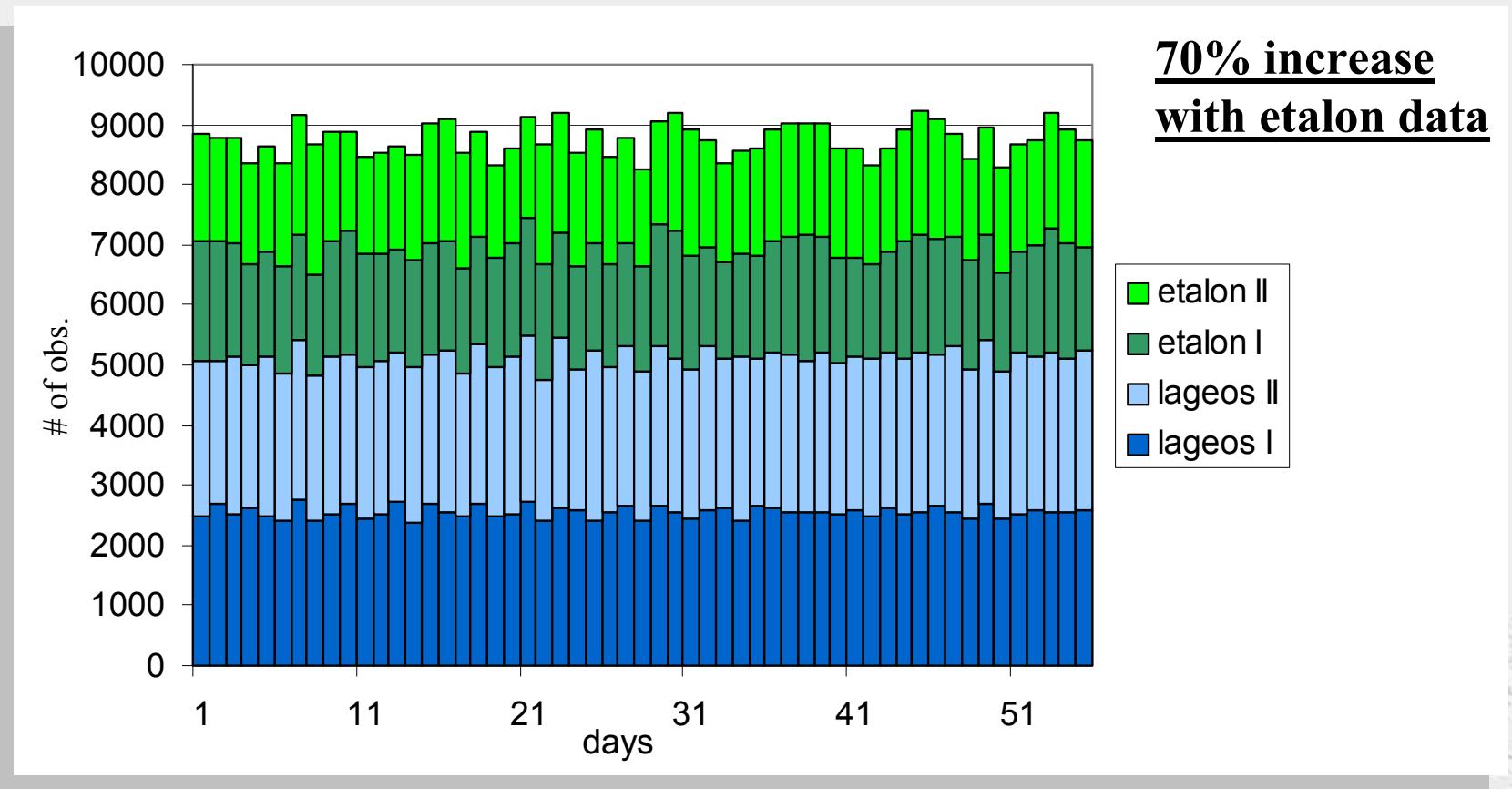
The DOP factor decreases with \sqrt{n} , being n the number of observation.
A normalized value of the DOP factor does not contain the effect of the
data increase

$$\text{norm_DOP} = \text{DOP} \cdot \sqrt{\frac{\text{obs}(L+E)}{\text{obs}(L)}}$$



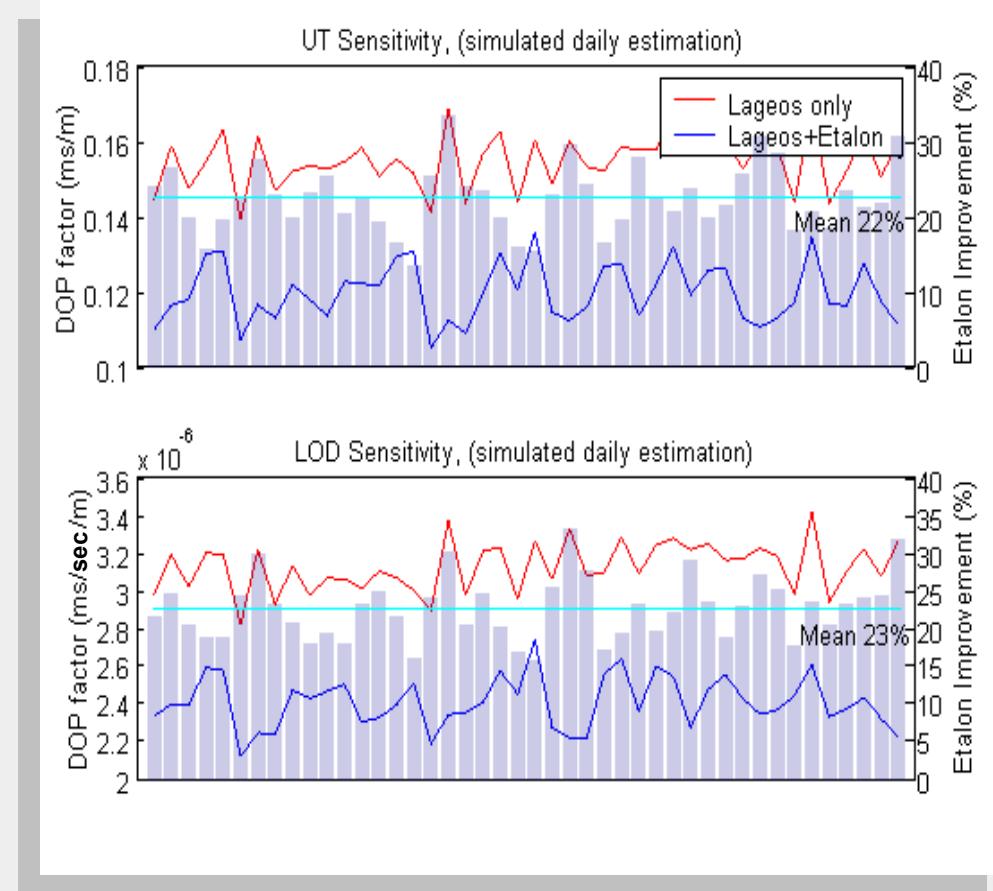
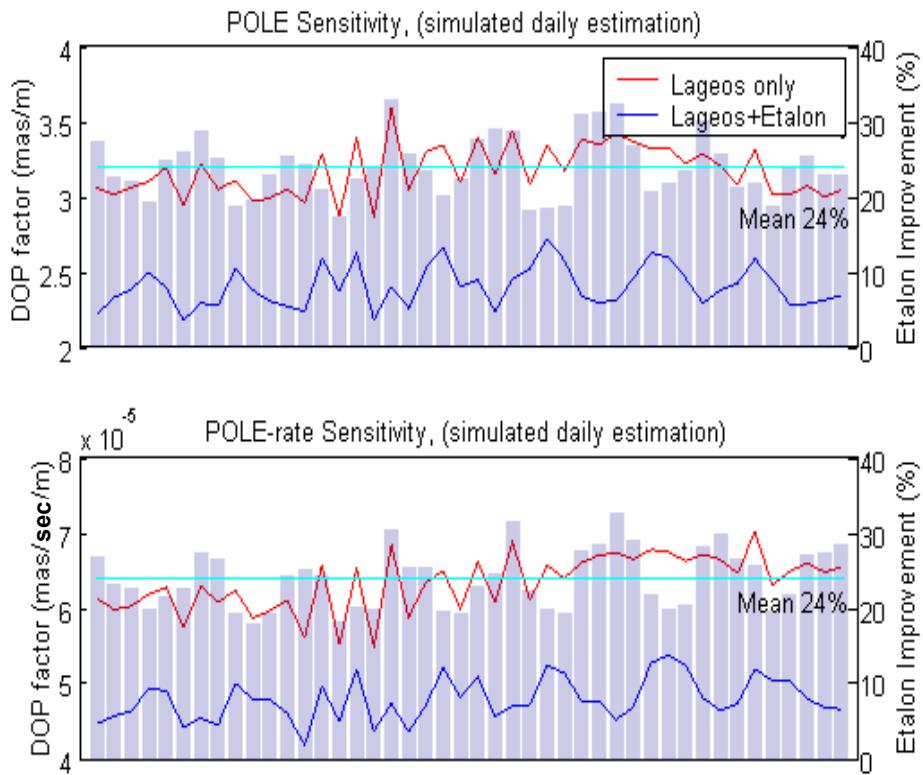
SIMULATION

- real observing network
- 2 minutes sampling for Lageos, 5 for Etalon
- all passes, no pass conflict at the station considered

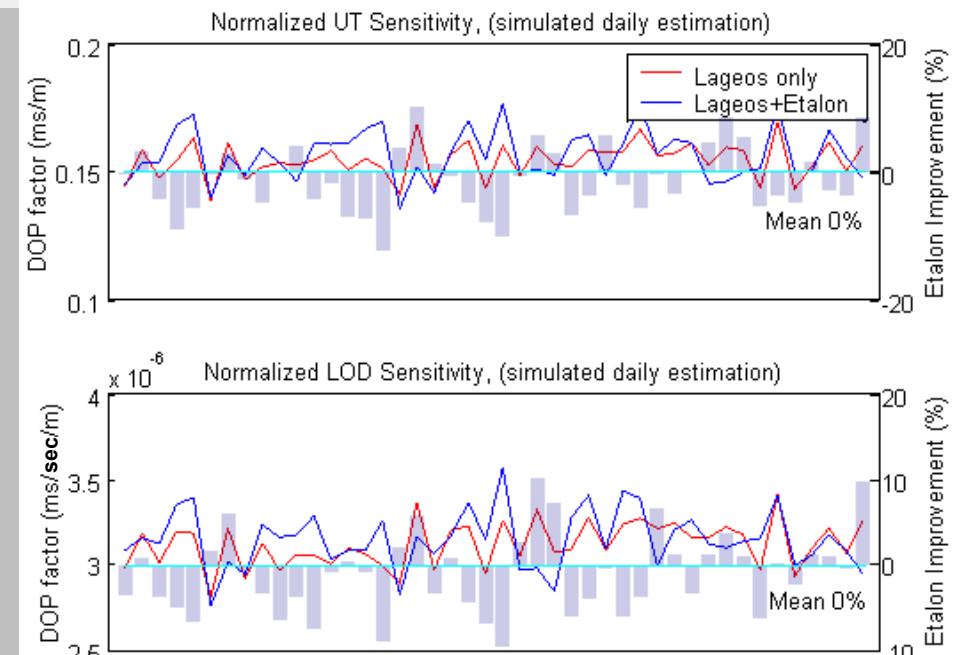
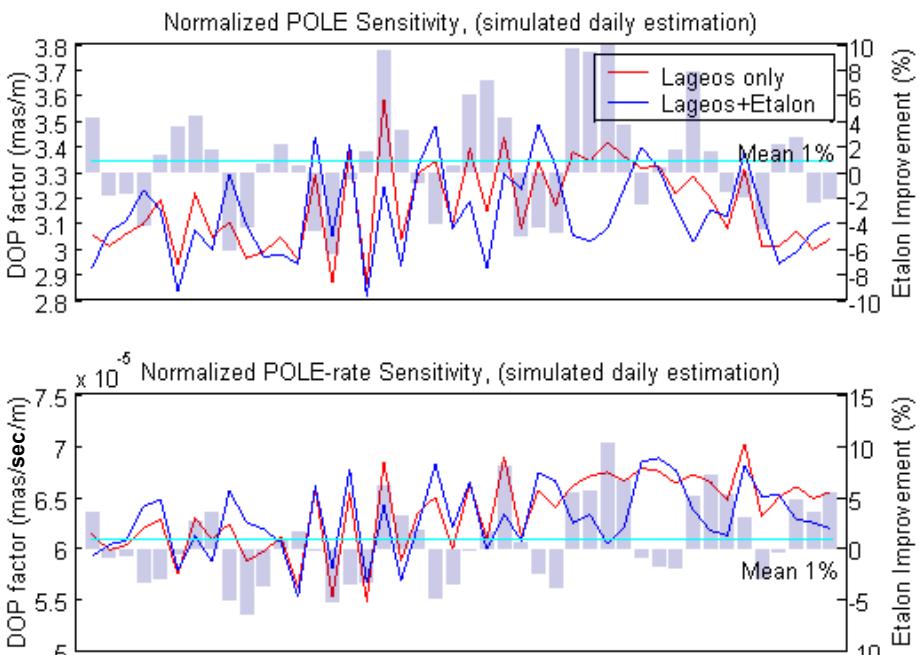




The EOP DOP from simulated data



The normalized EOP DOP from simulated data



Optimum data weighting

Following Lerch (1989), the calibration factor for each data set t is defined by:

$$k_t = \frac{(X_t - X)^2}{\sigma_t^2 - \sigma^2}$$

where:

X is the solution EOP vector for the complete satellite constellation

X_t is the solution EOP vector without considering the satellite ‘ t ’

σ and σ_t the vectors of the sigmas of the two solutions

Starting from an initial weight w_t , the adjusted weight is:

$$\vec{w}_t = w_t / k_t$$

The final weight to be applied at the solution t will be obtained by iterating the solution X_t , for each data set t , until $k_t=1$

'Weighted' solution



EOP

	same weights		optimum weights	
	wrms *	σ	wrms *	σ
X (mas)	0.64	0.18	0.63	0.18
Y (mas)	0.57	0.17	0.56	0.17
UT (ms)	0.05	0.02	0.05	0.02

EOP rate

	same weights		optimum weights	
	wrms *	σ	wrms *	σ
Xr (mas/d)	1.06	0.30	0.98	0.28
Yr (mas/d)	1.09	0.30	1.06	0.30
LOD (ms/d)	0.08	0.02	0.07	0.02

* wrms of the residuals with respect to EOPC04

Mean satellites weights

L1	L2	ET1	ET2
1.0	0.2	0.8	0.7





Summary

Very small improvement from the Etalon satellites seen by:

- analysis of real data
 - analysis of simulated data
 - DOP
 - optimum data weighting
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